

Renormalization group evolution of collinear and infrared divergences

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- Higher-order collinear and soft corrections
- Factorization, RGE, and Resummation
- Two-loop eikonal calculations
- Soft anomalous dimensions
- $t\bar{t}$ and single top production
- H^- , W^- , and γ production

Higher-order collinear and soft corrections

Soft-gluon corrections from incomplete cancellations of infrared divergences between virtual diagrams and real diagrams with soft (low-energy) gluons

Soft corrections $\left[\frac{\ln^k(s_4/m^2)}{s_4} \right]_+$ with $k \leq 2n - 1$ and s_4 distance from threshold

double collinear and soft logarithms

also purely collinear terms $\frac{1}{m^2} \ln^k(s_4/m^2)$

Soft-gluon corrections are dominant near threshold

Resum (exponentiate) these corrections

At NLL accuracy requires one-loop calculations in the eikonal approximation

Recent results at NNLL – two-loop calculations completed

Approximate NNLO cross section from expansion of resummed cross section

Essential ingredient: two-loop soft anomalous dimension

Allows NNLL resummation

Factorization, RGE, and Resummation

Resummation follows from factorization properties of the cross section

- performed in moment space

$$\sigma = (\prod \psi) H_{IL} S_{LI} (\prod J) \quad H: \text{hard-scattering function} \quad S: \text{soft-gluon function}$$

Use RGE to evolve function associated with soft-gluon emission

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g_s) \frac{\partial}{\partial g_s} \right) S_{LI} = -(\Gamma_S^\dagger)_{LB} S_{BI} - S_{LA} (\Gamma_S)_{AI}$$

Γ_S is the soft anomalous dimension - a matrix in color space and a function of kinematical invariants s, t, u

Resummed cross section

$$\begin{aligned} \hat{\sigma}^{res}(N) &= \exp \left[\sum_i E_i(N_i) \right] \exp \left[\sum_j E'_j(N') \right] \exp \left[\sum_{i=1,2} 2 \int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu} \gamma_{q/q} (\tilde{N}_i, \alpha_s(\mu)) \right] \\ &\times \text{tr} \left\{ H(\alpha_s) \exp \left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu} \Gamma_S^\dagger(\alpha_s(\mu)) \right] S \left(\alpha_s \left(\frac{\sqrt{s}}{\tilde{N}'} \right) \right) \exp \left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu} \Gamma_S(\alpha_s(\mu)) \right] \right\} \end{aligned}$$

collinear and soft radiation from incoming partons

$$E_i(N_i) = \int_0^1 dz \frac{z^{N_i-1} - 1}{1-z} \left\{ \int_{(1-z)^2}^{(1-z)^2} \frac{d\lambda}{\lambda} A_i(\alpha_s(\lambda s)) + D_i[\alpha_s((1-z)^2 s)] \right\}$$

purely collinear: replace $\frac{z^{N-1}-1}{1-z}$ by $-z^{N-1}$

collinear and soft radiation from outgoing massless quarks and gluons

$$E'(N') = \int_0^1 dz \frac{z^{N'-1} - 1}{1-z} \left\{ \int_{(1-z)^2}^{1-z} \frac{d\lambda}{\lambda} A_i(\alpha_s(\lambda s)) + B_i[\alpha_s((1-z)s)] + D_i[\alpha_s((1-z)^2 s)] \right\}$$

factorization scale μ_F dependence controlled by

$$\gamma_{i/i} = -A_i \ln \tilde{N}_i + \gamma_i$$

Noncollinear soft gluon emission controlled by the soft anomalous dimension Γ_S

determine Γ_S from coefficients of ultraviolet poles in dimensionally regularized eikonal diagrams

Eikonal approximation

Feynman rules for soft gluon emission simplify

$$\bar{u}(p) (-ig_s T_F^c) \gamma^\mu \frac{i(p+k+m)}{(p+k)^2 - m^2 + i\epsilon} \rightarrow \bar{u}(p) g_s T_F^c \gamma^\mu \frac{p+m}{2p \cdot k + i\epsilon} = \bar{u}(p) g_s T_F^c \frac{v^\mu}{v \cdot k + i\epsilon}$$

with $p \propto v$, T_F^c generators of SU(3)

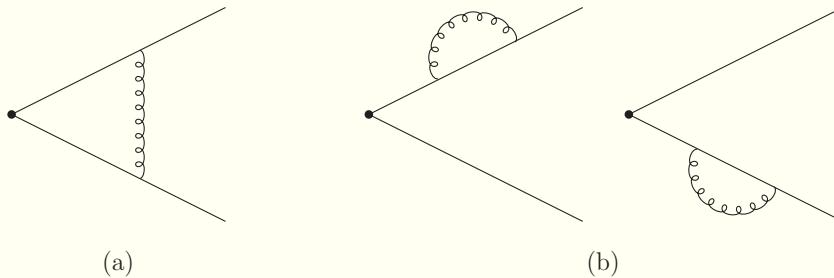
Perform calculation in momentum space and Feynman gauge

Complete two-loop results for

- soft (cusp) anomalous dimension for $e^+e^- \rightarrow t\bar{t}$
- $t\bar{t}$ hadroproduction
- t -channel single top production
- s -channel single top production
- $bg \rightarrow tW^-$ and $bg \rightarrow tH^-$
- direct photon and W production at large Q_T

Soft (cusp) anomalous dimension

One-loop eikonal diagrams



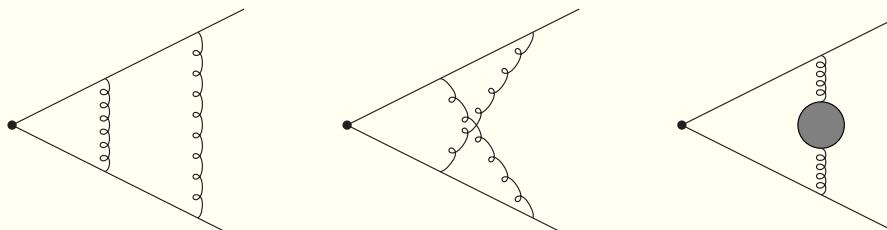
$$\Gamma_S = \frac{\alpha_s}{\pi} \Gamma_S^{(1)} + \frac{\alpha_s^2}{\pi^2} \Gamma_S^{(2)} + \dots$$

The one-loop soft anomalous dimension, $\Gamma_S^{(1)}$, can be read off the coefficient of the ultraviolet (UV) pole of the one-loop diagrams

$$\Gamma_S^{(1)} = C_F \left[-\frac{(1+\beta^2)}{2\beta} \ln \left(\frac{1-\beta}{1+\beta} \right) - 1 \right] \quad \text{with} \quad \beta = \sqrt{1 - \frac{4m^2}{s}}$$

Two-loop eikonal diagrams

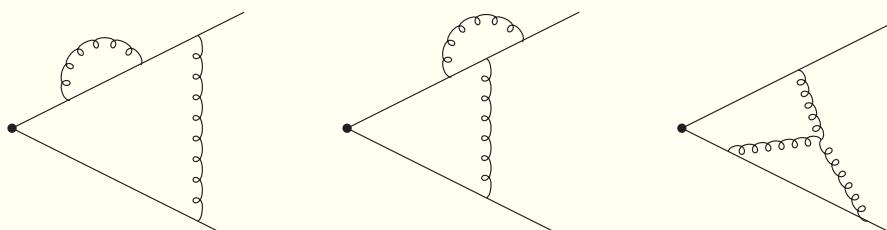
Vertex correction graphs



(a)

(b)

(c)

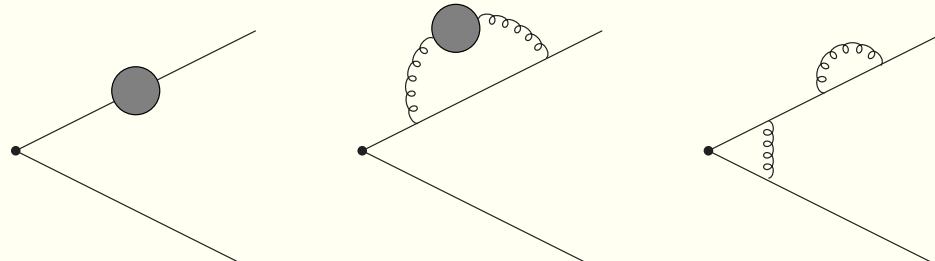


(d)

(e)

(f)

Heavy-quark self-energy graphs



(a)

(b)

(c)

Include counterterms for all graphs and multiply with corresponding color factors

Determine two-loop soft anomalous dimension from UV poles of the sum of the graphs

$$\begin{aligned} \Gamma_S^{(2)} = & \frac{K}{2} \Gamma_S^{(1)} + C_F C_A M_\beta = \frac{K}{2} \Gamma_S^{(1)} + C_F C_A \left\{ \frac{1}{2} + \frac{\zeta_2}{2} + \frac{1}{2} \ln^2 \left(\frac{1-\beta}{1+\beta} \right) \right. \\ & - \frac{(1+\beta^2)^2}{8\beta^2} \left[\zeta_3 + \zeta_2 \ln \left(\frac{1-\beta}{1+\beta} \right) + \frac{1}{3} \ln^3 \left(\frac{1-\beta}{1+\beta} \right) + \ln \left(\frac{1-\beta}{1+\beta} \right) \text{Li}_2 \left(\frac{(1-\beta)^2}{(1+\beta)^2} \right) - \text{Li}_3 \left(\frac{(1-\beta)^2}{(1+\beta)^2} \right) \right] \\ & - \frac{(1+\beta^2)}{4\beta} \left[\zeta_2 - \zeta_2 \ln \left(\frac{1-\beta}{1+\beta} \right) + \ln^2 \left(\frac{1-\beta}{1+\beta} \right) - \frac{1}{3} \ln^3 \left(\frac{1-\beta}{1+\beta} \right) + 2 \ln \left(\frac{1-\beta}{1+\beta} \right) \ln \left(\frac{(1+\beta)^2}{4\beta} \right) \right. \\ & \left. \left. - \text{Li}_2 \left(\frac{(1-\beta)^2}{(1+\beta)^2} \right) \right] \right\} \end{aligned}$$

where $K = C_A (67/18 - \zeta_2) - 5n_f/9$

N. Kidonakis, Phys. Rev. Lett. 102, 232003 (2009), arXiv:0903.2561 [hep-ph]

$\Gamma_S^{(2)}$ vanishes at $\beta = 0$, the threshold limit, and diverges at $\beta = 1$, the massless limit

If one quark is massless and one is massive

$$\Gamma_S^{(2)} = \frac{K}{2} \Gamma_S^{(1)} + C_F C_A \frac{(1-\zeta_3)}{4}$$

QCD processes: Color structure gets more complicated with more than two colored partons in the process - Cusp anomalous dimension an essential component of other calculations

Top-antitop production in hadron colliders

The soft anomalous dimension matrix for $q\bar{q} \rightarrow t\bar{t}$ is

$$\Gamma_{S q\bar{q}} = \begin{bmatrix} \Gamma_{q\bar{q}11} & \Gamma_{q\bar{q}12} \\ \Gamma_{q\bar{q}21} & \Gamma_{q\bar{q}22} \end{bmatrix}$$

At one loop

$$\begin{aligned} \Gamma_{q\bar{q}11}^{(1)} &= -C_F [L_\beta + 1] & \Gamma_{q\bar{q}21}^{(1)} &= 2 \ln \left(\frac{u_1}{t_1} \right) & \Gamma_{q\bar{q}12}^{(1)} &= \frac{C_F}{C_A} \ln \left(\frac{u_1}{t_1} \right) \\ \Gamma_{q\bar{q}22}^{(1)} &= C_F \left[4 \ln \left(\frac{u_1}{t_1} \right) - L_\beta - 1 \right] + \frac{C_A}{2} \left[-3 \ln \left(\frac{u_1}{t_1} \right) + \ln \left(\frac{t_1 u_1}{sm^2} \right) + L_\beta \right] \end{aligned}$$

where $L_\beta = \frac{1+\beta^2}{2\beta} \ln \left(\frac{1-\beta}{1+\beta} \right)$ with $\beta = \sqrt{1 - 4m^2/s}$

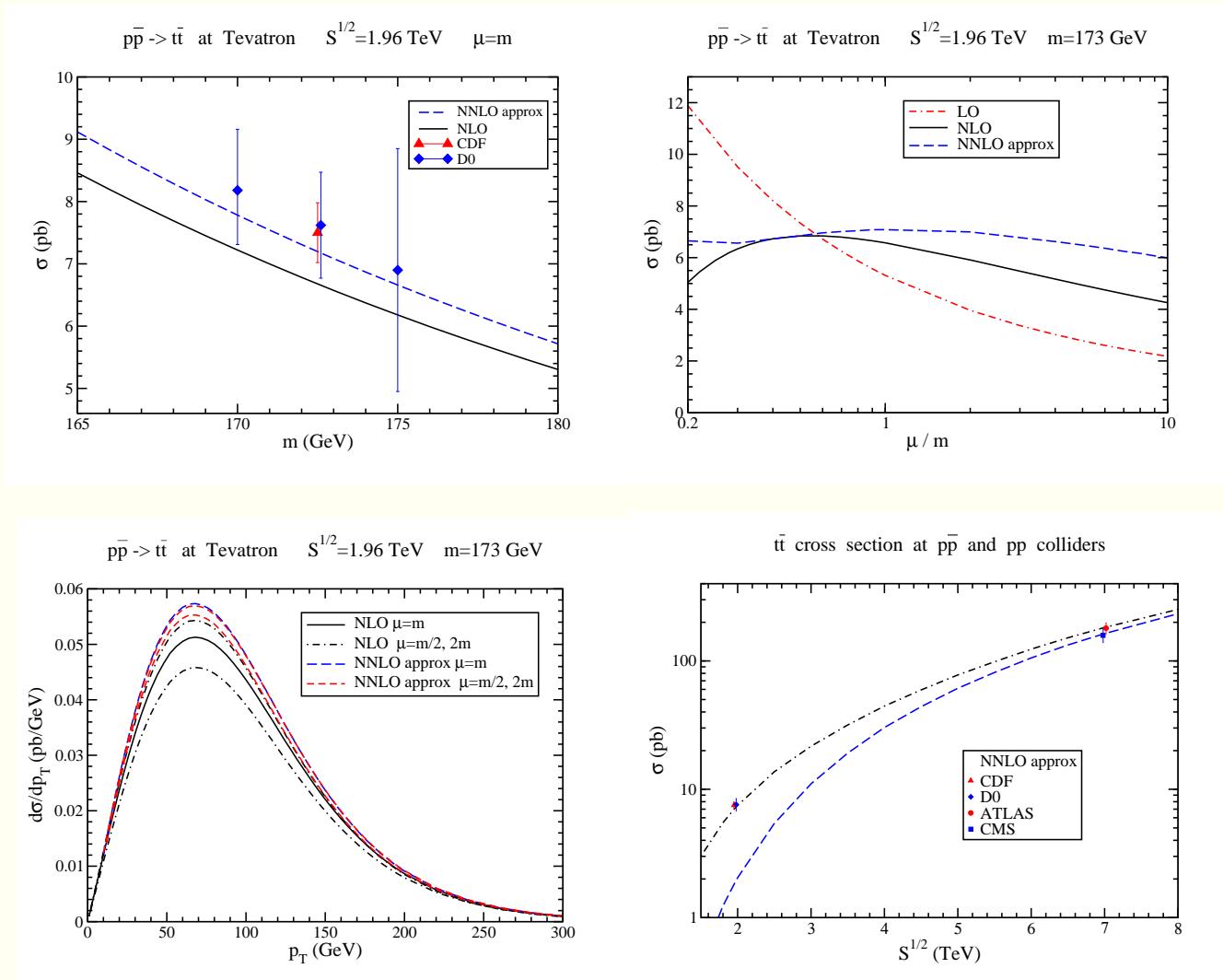
Write the two-loop cusp anomalous dimension as $\Gamma_S^{(2)} = \frac{K}{2} \Gamma_S^{(1)} + C_F C_A M_\beta$. Then at two loops

$$\begin{aligned} \Gamma_{q\bar{q}11}^{(2)} &= \frac{K}{2} \Gamma_{q\bar{q}11}^{(1)} + C_F C_A M_\beta & \Gamma_{q\bar{q}22}^{(2)} &= \frac{K}{2} \Gamma_{q\bar{q}22}^{(1)} + C_A \left(C_F - \frac{C_A}{2} \right) M_\beta \\ \Gamma_{q\bar{q}21}^{(2)} &= \frac{K}{2} \Gamma_{q\bar{q}21}^{(1)} + C_A N_\beta \ln \left(\frac{u_1}{t_1} \right) & \Gamma_{q\bar{q}12}^{(2)} &= \frac{K}{2} \Gamma_{q\bar{q}12}^{(1)} - \frac{C_F}{2} N_\beta \ln \left(\frac{u_1}{t_1} \right) \end{aligned}$$

with N_β a subset of terms of M_β

Similar results for $gg \rightarrow t\bar{t}$ channel N. Kidonakis, Phys. Rev. D 82, 114030 (2010), arXiv:1009.4935 [hep-ph]

$t\bar{t}$ cross section at the Tevatron and LHC



Single top quark production - t channel

Dominant single top production channel at both Tevatron and LHC energies

Soft anomalous dimension for t -channel single top production

One loop

$$\Gamma_{S11}^{(1)} = C_F \left[\ln\left(\frac{-t}{s}\right) + \ln\left(\frac{m_t^2 - t}{m_t \sqrt{s}}\right) - \frac{1}{2} \right]$$

$$\Gamma_{S21}^{(1)} = \ln\left(\frac{u(u - m_t^2)}{s(s - m_t^2)}\right)$$

$$\Gamma_{S12}^{(1)} = \frac{C_F}{2N_c} \Gamma_{S21}^{(1)}$$

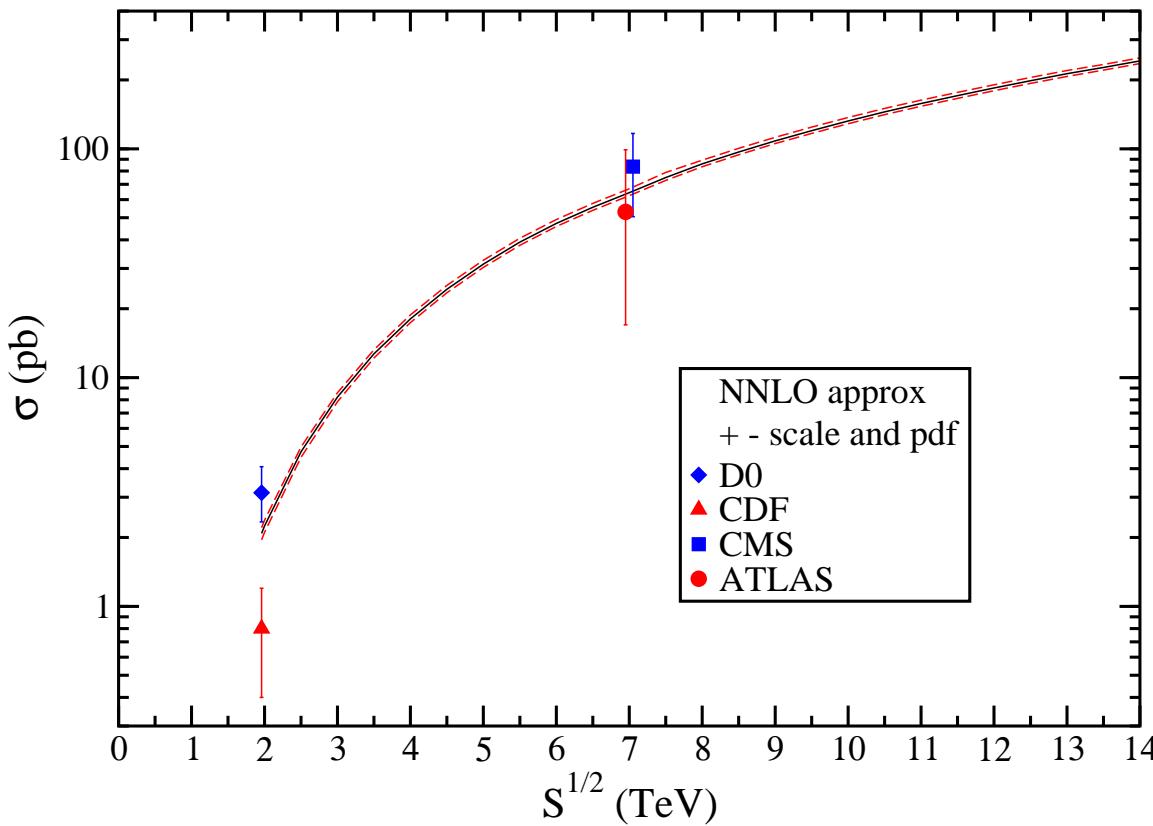
Two loops

$$\Gamma_{S11}^{(2)} = \frac{K}{2} \Gamma_{S11}^{(1)} + C_F C_A \frac{(1 - \zeta_3)}{4}$$

N. Kidonakis, Phys. Rev. D (in press), arXiv:1103.2792 [hep-ph]

t-channel combined cross section versus energy

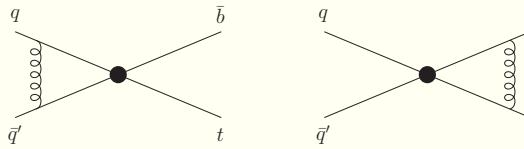
t-channel single top + single antitop cross section



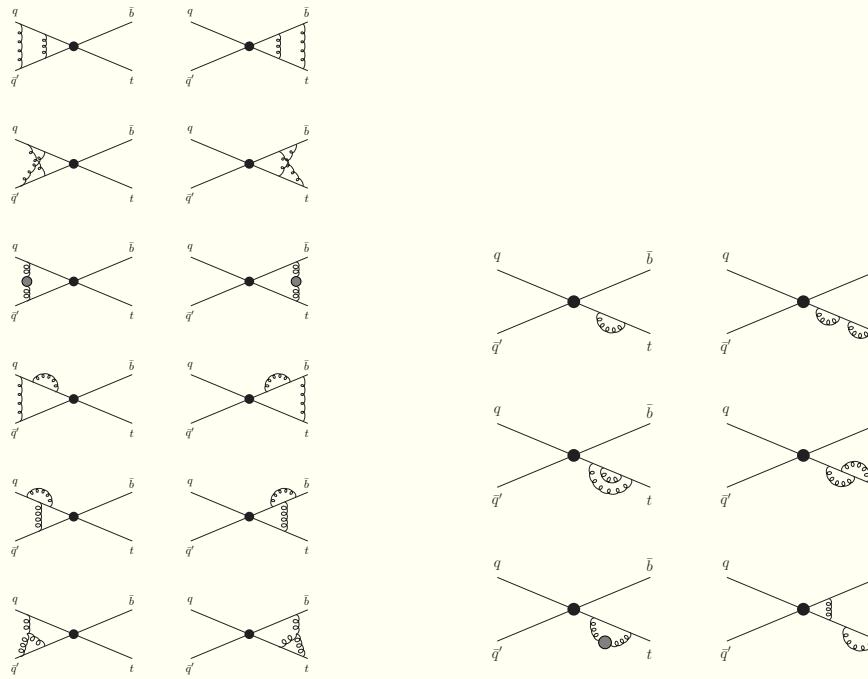
Tevatron and LHC results are consistent with theory

Single top quark production - s channel

One-loop eikonal diagrams



Two-loop eikonal diagrams



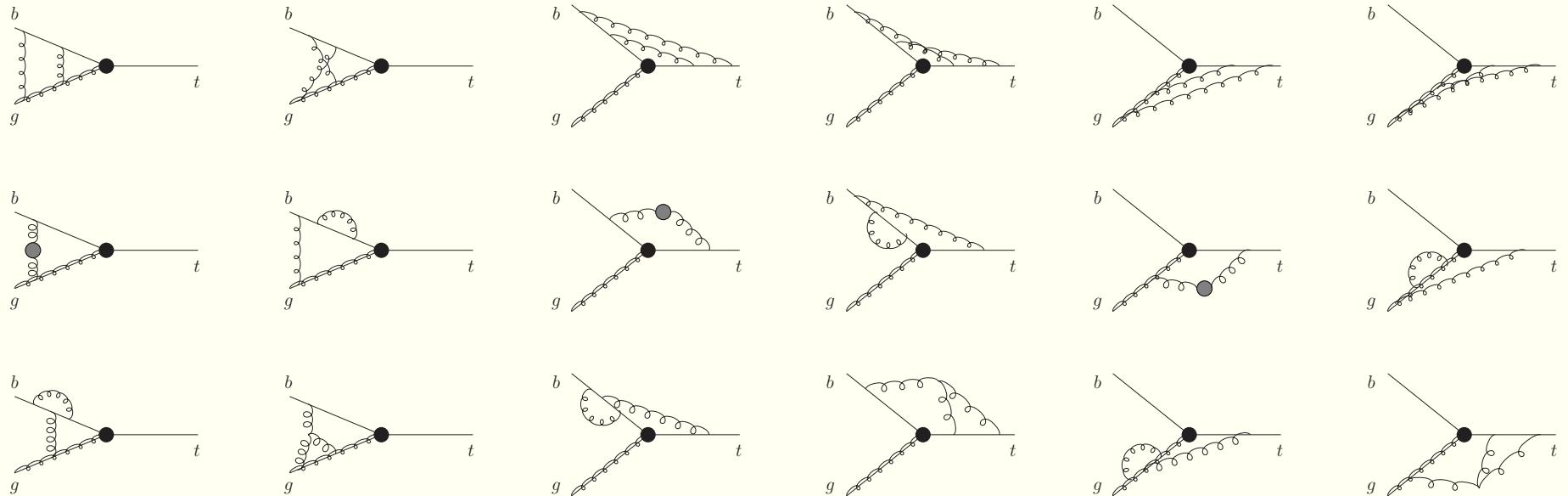
Soft anomalous dimension for s -channel single top production

$$\Gamma_{S11}^{(1)} = C_F \left[\ln \left(\frac{s - m_t^2}{m_t \sqrt{s}} \right) - \frac{1}{2} \right], \quad \Gamma_{S11}^{(2)} = \frac{K}{2} \Gamma_{S11}^{(1)} + C_F C_A \frac{(1 - \zeta_3)}{4}$$

N. Kidonakis, Phys. Rev. D 81, 054028 (2010), arXiv:1001.5034 [hep-ph]

Associated production of a top quark with a W^- or H^-

Two-loop eikonal diagrams (+ extra top-quark self-energy graphs)



Soft anomalous dimension for $bg \rightarrow tW^-$ (or $bg \rightarrow tH^-$)

$$\Gamma_{S,tW^-}^{(1)} = C_F \left[\ln \left(\frac{m_t^2 - t}{m_t \sqrt{s}} \right) - \frac{1}{2} \right] + \frac{C_A}{2} \ln \left(\frac{m_t^2 - u}{m_t^2 - t} \right)$$

$$\Gamma_{S,tW^-}^{(2)} = \frac{K}{2} \Gamma_{S,tW^-}^{(1)} + C_F C_A \frac{(1 - \zeta_3)}{4}$$

N. Kidonakis, Phys. Rev. D 82, 054018 (2010), arXiv:1005.4451 [hep-ph]

Direct photon and W -boson production at large Q_T

Threshold corrections dominate at large transverse momentum

Two loop soft anomalous dimensions for NNLL resummation

For $qg \rightarrow Wq$ or $qg \rightarrow \gamma q$

$$\Gamma_{S, qg \rightarrow Wq}^{(1)} = C_F \ln \left(\frac{-u}{s} \right) + \frac{C_A}{2} \ln \left(\frac{t}{u} \right)$$

$$\Gamma_{S, qg \rightarrow Wq}^{(2)} = \frac{K}{2} \Gamma_{S, qg \rightarrow Wq}^{(1)}$$

For $q\bar{q} \rightarrow Wg$ or $q\bar{q} \rightarrow \gamma g$

$$\Gamma_{S, q\bar{q} \rightarrow Wg}^{(1)} = \frac{C_A}{2} \ln \left(\frac{tu}{s^2} \right)$$

$$\Gamma_{S, q\bar{q} \rightarrow Wg}^{(2)} = \frac{K}{2} \Gamma_{S, q\bar{q} \rightarrow Wg}^{(1)}$$

Summary

- Collinear and soft logarithms
- Factorization, RGE, and resummation
- Two-loop soft anomalous dimension matrices
- NNLL resummation for top quark production
and other processes